Production of pseudovector ($J^{PC} = 1^{++}$) heavy quarkonia by virtual Z boson in e^+e^- collisions *

N.N. Achasov

Laboratory of Theoretical Physics

S.L. Sobolev Institute for Mathematics

Universitetskii prospekt, 4

Novosibirsk 90, 630090, Russia †

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Abstract

It is shown that $BR(\chi_{b1}(1P) \to Z \to e^+e^-) \simeq 3.3 \cdot 10^{-7}$, $BR(\chi_{b1}(2P) \to Z \to e^+e^-) \simeq 4.1 \cdot 10^{-7}$ and $BR(\chi_{c1}(1P) \to Z \to e^+e^-) \simeq 10^{-8}$ that give a good chance to search for the direct production of pseudovector $^3P_1$ heavy quarkonia in e^+e^- collisions ($e^+e^- \to Z \to^3 P_1$) even at current facilities not to mention b and $c-\tau$ factories.

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†E-mail: achasov@math.nsk.su

Broadly speaking, production of narrow p wave pseudovector bound states of heavy quarks ${}^3P_1$ with $J^{PC} = 1^{++}$, like $\chi_{b1}(1p)$, $\chi_{b1}(2p)$ and $\chi_{c1}(1p)$ [1], in e^+e^- collisions via the virtual intermediate Z boson ($e^+e^- \to Z \to^3 P_1$), could be observed by experiment at least at b and $c-\tau$ factories for amplitudes of weak interactions grow with energy increase in this energy regions $\propto G_F E^2$.

In present paper it is shown that the experimental investigation of this interesting phenomenon is possible at current facilities.

First and foremost let us calculate the ${}^3P_1 \to Z \to e^+e^-$ amplitude. We use a handy formalism of description of a nonrelativistic bound state decays given in the review [2].

The Feynman amplitude describing a free quark-antiquark annihilation into the e^+e^- pair has the form

$$M(\bar{Q}(p_{\bar{Q}})Q(p_Q) \to Z \to e^+(p_+)e^-(p_-)) = \frac{\alpha\pi}{2\cos^2\theta_W \sin^2\theta_W} \frac{1}{E^2 - m_Z^2} j^{e\,\alpha} j^Q_\alpha =$$
$$\frac{\alpha\pi}{2\cos^2\theta_W \sin^2\theta_W} \frac{1}{E^2 - m_Z^2} \bar{e}(p_-)[(-1 + 4\sin^2\theta_W)\gamma^\alpha - \gamma^\alpha\gamma_5]e(-p_+) \cdot$$
$$\bar{Q}_C(-p_{\bar{Q}})[(t_3 - e_Q \sin^2\theta_W)\gamma_\alpha + t_3\gamma_\alpha\gamma_5]Q^C(p_Q) \qquad (1)$$

where the notation is quite marked unless generally accepted.

One can ignore the vector part of the electroweak electron-positron current $j^{e\,\alpha}$ in Eq. (1) for $(1 - 4\sin^2\theta_W) \simeq 0.1$. As for the electroweak quark-antiquark current j^Q_α, only its axial-vector part contributes to the pseudovector (1^{++}) quarkonia annihilation. So, the amplitude of interest is

$$M(\bar{Q}(p_{\bar{Q}})Q(p_Q) \to Z \to e^+(p_+)e^-(p_-)) = \sigma_Q \frac{\alpha\pi}{4\cos^2\theta_W \sin^2\theta_W} \frac{1}{m_Z^2} j^{e\,\alpha}_5 j^Q_{\alpha\,5} =$$
$$\sigma_Q \frac{\alpha\pi}{4\cos^2\theta_W \sin^2\theta_W} \frac{1}{m_Z^2} \bar{e}(p_-)\gamma^\alpha\gamma_5 e(-p_+)\bar{Q}_C(-p_{\bar{Q}})\gamma_\alpha\gamma_5 Q^C(p_Q) \qquad (2)$$

where $\sigma_c = 1$ and $\sigma_b = -1$. The term of order of E^2/m_Z^2 is omitted in Eq. (2).

In the c.m. system one can write that

$$M(\bar{Q}(p_{\bar{Q}})Q(p_Q) \to Z \to e^+(p_+)e^-(p_-)) \simeq -\sigma_Q \frac{\alpha\pi}{4\cos^2\theta_W \sin^2\theta_W} \frac{1}{m_Z^2} j^e_{i\,5}\, j^Q_{i\,5} =$$
$$-\sigma_Q \frac{\alpha\pi}{4\cos^2\theta_W \sin^2\theta_W} \frac{1}{m_Z^2} \bar{e}(p_-)\gamma_i\gamma_5 e(-p_+)\bar{Q}_C(-p_{\bar{Q}})\gamma_i\gamma_5 Q^C(p_Q) = M(\mathbf{p}) \qquad (3)$$

ignoring the term of order of $2m_e/E$ ($j^e_{0\,5} = (2m_e/E)j_5$ in the c.m. system). Hereafter the three-momentum $\mathbf{p} = \mathbf{p_Q} = -\mathbf{p_{\bar{Q}}}$ in the c.m. system.

To construct the effective Hamiltonian for the 1^{++} quarkonium annihilation into the e^+e^- pair one expresses the axial-vector quark-antiquark current $j^Q_{i\,5}$ in Eq. (3) in terms of two-component spinors of quark w^α and antiquark v_β using four-component Dirac bispinors

$$\begin{aligned} Q^C(p_Q) &= Q^C Q(p_Q) = \frac{1}{\sqrt{2m_Q}}\, Q^C \begin{pmatrix} \sqrt{\varepsilon + m_Q}\; w \\ \sqrt{\varepsilon - m_Q}\; (\mathbf{n}\cdot\boldsymbol{\sigma})w \end{pmatrix}, \\ \bar{Q}_C(-p_{\bar{Q}}) &= Q_C \bar{Q}(-p_{\bar{Q}}) = -\frac{1}{\sqrt{2m_Q}}\, Q_C \left(\sqrt{\varepsilon - m_Q}\; v(\mathbf{n}\cdot\boldsymbol{\sigma}),\, \sqrt{\varepsilon + m_Q}\; v\right) \end{aligned} \tag{4}$$

where Q^C and Q_C are color spinors of quark and antiquark respectively, $\mathbf{n} = \mathbf{p}/|\mathbf{p}|$.

As the result

$$j^Q_{i\,5} = \imath \frac{2\sqrt{6}}{2m_Q}\, \varepsilon_{kin} p_k \chi_n \eta_0 \tag{5}$$

where $\chi_n = v\sigma_n w/\sqrt{2}$ and $\eta_0 = Q_C Q^C/\sqrt{3}$.

The spin-factor χ_i and the color spin-factor η_0 are contracted as follows $\chi_i\chi_j = \delta_{ij}$ and $\eta_0\eta_0 = 1$.

The $^3P_1$ bound state wave function in the coordinate representation has the form

$$\Psi_j(^3P_1\,,\, \boldsymbol{r}\,,\, m_A) = \frac{1}{\sqrt{2}}\, \eta_0 \varepsilon_{jpl} \chi_p \frac{r_l}{r} \sqrt{\frac{3}{4\pi}}\, R_P(r\,,\, m_A) \tag{6}$$

where $r = |\mathbf{r}|$, $R_P(r, m_A)$ is a radial wave function with the normalization $\int_0^\infty |R_P(r\,,\, m_A)|^2 r^2 dr = 1$, m_A is a mass of a $^3P_1$ bound state.

For use one needs the $^3P_1$ bound state wave function in the momentum representation. It has the form

$$\Psi_j(^3P_1\,,\, \boldsymbol{p}\,,\, m_A) = \frac{1}{\sqrt{2}}\, \eta_0 \varepsilon_{jpl} \chi_p (\psi_P(\boldsymbol{p}\,,\, m_A))_l \tag{7}$$

where

$$(\psi_P(\boldsymbol{p}\,,\, m_A))_l = \sqrt{\frac{3}{4\pi}} \int \frac{r_l}{r} R_P(r\,,\, m_A) \exp\{-\imath(\boldsymbol{p}\cdot\boldsymbol{r})\} d^3r\,. \tag{8}$$

The amplitude of the $^3P_1$ bound state $\to e^+e^-$ annihilation is given by

$$M(A_j \to e^+e^-) \equiv \int M(\boldsymbol{p})\Psi_j(^3P_1\,,\,\boldsymbol{p}\,,\,m_A)\frac{d^3p}{(2\pi)^3} \tag{9}$$

where A_j stands for a $^3P_1$ state.

As seen from Eq. (3) to find the $M(A_j \to e^+e^-)$ amplitude one needs to calculate the convolution and contraction of a quark-antiquark pair axial-vector current with a $^3P_1$ bound state wave function

$$\int j^Q_{i5}\Psi_j(^3P_1\,,\,\boldsymbol{p}\,,\,m_A)\frac{d^3p}{(2\pi)^3} = \imath\frac{2\sqrt{6}}{m_A}\,\varepsilon_{kin}\chi_n\eta_0\int p_k\cdot\Psi_j(^3P_1\,,\,\boldsymbol{p}\,,\,m_A)\frac{d^3p}{(2\pi)^3} =$$
$$\imath\delta_{ij}\frac{4}{\sqrt{3}}\frac{1}{m_A}\int p_k(\psi_P(\boldsymbol{p}\,,\,m_A))_k\frac{d^3p}{(2\pi)^3} = \delta_{ij}2\frac{3}{\sqrt{\pi}}\frac{1}{m_A}R'_P(0\,,\,m_A) \tag{10}$$

where $R'_P(0\,,\,m_A) = dR_P(r\,,\,m_A)/dr|_{r=0}$. Deriving Eq. (10) we put, as it usually is, $2m_Q = m_A$ and took into account that $R_P(r\,,\,m_A) \to rR'_P(0\,,\,m_A)$ when $r \to 0$.

So,

$$M(A_j \to e^+e^-) = -\sigma_Q 3\sqrt{\pi}\,\frac{\alpha}{2\cos^2\theta_W\sin^2\theta_W}\frac{1}{m_Z^2}\frac{1}{m_A}R'_P(0)\bar{e}(p_-)\gamma_j\gamma_5 e(-p_+)\,. \tag{11}$$

The width of the $A \to e^+e^-$ decay

$$\Gamma(A \to e^+e^-) = \frac{1}{3}\sum_{j\,e^+e^-}\int |M(A_j \to e^+e^-)|^2(2\pi)^4\delta^4(m_A - p_- - p_+)\frac{d^3p_+}{(2\pi)^3}\frac{d^3p_-}{(2\pi)^3} \simeq$$
$$\frac{1}{3}\sum_{j\,e^+e^-}\int |M(A_j \to e^+e^-)|^2\frac{1}{8\pi} \simeq$$
$$\frac{\alpha^2}{32}\frac{3}{\cos^4\theta_W\sin^4\theta_W}\frac{1}{m_Z^4}\frac{1}{m_A^2}|R'_P(0\,,\,m_A)|^2 Sp(\hat{p}_+\gamma_j\gamma_5\hat{p}_-\gamma_j\gamma_5) \simeq$$
$$\frac{\alpha^2}{8}\frac{1}{\cos^4\theta_W\sin^4\theta_W}\frac{1}{m_Z^4}|R'_P(0\,,\,m_A)|^2 \simeq 12.3\alpha^2\frac{1}{m_Z^4}|R'_P(0\,,\,m_A)|^2 \tag{12}$$

where terms of order of $(2m_e/m_A)^2$ are omitted, $\sin^2\theta_W = 0.225$ is put, the normalization $\bar{e}(p_-)e(p_-) = 2m_e$ and $\bar{e}(-p_+)e(-p_+) = -2m_e$ is used. Note that for the quark and antiquark we use the normalization $\bar{Q}(p_Q)Q(p_Q) = 1$ and $\bar{Q}(-p_{\bar{Q}})Q(-p_{\bar{Q}}) = -1$, see Eq. (4).

To estimate a possibility of the $\chi_{c1}(1P)$, $\chi_{b1}(1P)$ and $\chi_{b1}(2P)$ production in e^+e^- collisions it needs to estimate the branching ratio $BR(A \to e^+e^-)$.

In a logarithmic approximation [2,3] the decay of the $^3P_1$ level into hadrons is caused by the decays $^3P_1 \to g + q\bar{q}$ where g is gluon and $q\bar{q}$ is a pair of light quarks: $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ for $\chi_{c1}(1P)$ and $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, $c\bar{c}$ for $\chi_{b1}(1P)$ and $\chi_{b1}(2P)$. The relevant width [2,3]

$$\Gamma_{log}\left(^3P_1 \equiv A \to gq\bar{q}\right) \simeq \frac{N}{3}\frac{128}{3\pi}\frac{\alpha_s^3}{m_A^4}|R'_P(0\,,\,m_A)|^2 \ln\frac{m_A R(m_A)}{2} \tag{13}$$

where N is the number of the light quark flavors and $R(m_A)$ is the quarkonium radius. Using Eqs. (12) and (13) one gets that

$$BR(A \to e^+e^-) \simeq \frac{3}{N}0.9\frac{\alpha^2}{\alpha_s^3}\left(\frac{m_A}{m_Z}\right)^4 \frac{1}{\ln\left(m_A R(m_A)/2\right)}\left[1 - \sum_V BR(A \to \gamma V)\right] \tag{14}$$

where the radiative decays $\chi_{c1}(1P) \to \gamma J/\psi$, $\chi_{b1}(1P) \to \gamma\Upsilon(1S)$, $\chi_{b1}(2P) \to \gamma\Upsilon(1S)$ and $\chi_{b1}(2P) \to \gamma\Upsilon(2S)$ are taken into account.

A convention [2] uses that $L(m_A) = \ln\left(m_A R(m_A)/2\right) \simeq 1$ for $\chi_{c1}(1P)$, i.e. when $m_A = 3.51\,GeV$ [1]. As for $\chi_{b1}(1P)$, $m_A = 9.89\,GeV$ [1], and $\chi_{b1}(2P)$, $m_A = 10.2552\,GeV$ [1], it depends on the m_A behavior of the quarkonium radius $R(m_A)$. For example, the coulomb-like potential gives that $R(m_A) \sim 1/m_A$ and the logarithm practically does not increase, $L(3.51\,GeV) \simeq L(9.89\,GeV) \simeq L(10.2552\,GeV) \simeq 1$. Alternatively, the harmonic oscillator potential gives $R(m_A) \sim 1/\sqrt{m_A\omega_0}$, where $\omega_0 \simeq 0.3\,GeV$, that leads to $L(9.89\,GeV) \simeq L(10.2552\,GeV) \simeq 1.5$. To be conservative one takes $L(9.89\,GeV) = L(10.2552\,GeV) = 2$.

So, putting $\alpha_s(3.51\,GeV) = 0.2\,,\, BR(\chi_{c1}(1P) \to \gamma J/\psi) = 0.27$ [1], $N = 3\,,\, L(3.51\,GeV) = 1\,,\, m_A = m_{\chi_{c1}(1P)} = 3.51\,GeV$ and $m_Z = 91.2\,GeV$ one gets from Eq. (14) that

$$BR(\chi_{c1}(1P) \to e^+e^-) = 0.96 \cdot 10^{-8}\,. \tag{15}$$

Putting $\alpha_s(9.89\,GeV) = \alpha_s(10.2552\,GeV) = 0.17\,,\, BR(\chi_{b1}(1P) \to \gamma\Upsilon(1S)) = 0.35$ [1] , $BR(\chi_{b1}(2P) \to \gamma\Upsilon(1S)) + BR(\chi_{b1}(2P) \to \gamma\Upsilon(2S)) = 0,085 + 0.21 = 0.295$ [1] , $N = 4\,,\, L(9.89\,GeV) = L(10.2552\,GeV) = 2\,,\, m_A = m_{\chi_{b1}(1P)} = 9.89\,GeV\,,\, m_A = m_{\chi_{b1}(2P)} = 10.2552\,GeV$ and $m_Z = 91.2\,GeV$ one gets from Eq. (14) that

$$BR(\chi_{b1}(1P) \to e^+e^-) = 3.3 \cdot 10^{-7}\,, \tag{16}$$

$$BR(\chi_{b1}(2P) \to e^+e^-) = 4.1 \cdot 10^{-7}\,.$$

Let us discuss possibilities to measure the branching ratios under consideration .

The cross section of a reaction $e^+e^- \to A \to out$ at resonance peak [1]

$$\sigma(A) \simeq 1.46 \cdot 10^{-26} BR(A \to e^+e^-) BR(A \to out) \left(\frac{GeV}{m_A}\right)^2 \, cm^2 \, . \tag{17}$$

So, for the $\chi_{c1}(1P)$ state production

$$\sigma\left(\chi_{c1}(1P)\right) \simeq 1.14 \cdot 10^{-35} BR\left(\chi_{c1}(1P) \to out\right) \, cm^2 \tag{18}$$

and for the production of the $\chi_{b1}(1P)$ and $\chi_{b1}(2P)$ states

$$\sigma\left(\chi_{b1}(1P)\right) \simeq 4.8 \cdot 10^{-35} BR\left(\chi_{b1}(1P) \to out\right) \, cm^2 \, , \tag{19}$$
$$\sigma\left(\chi_{b1}(2P)\right) \simeq 5.6 \cdot 10^{-35} BR\left(\chi_{b1}(2P) \to out\right) \, cm^2 \, .$$

In general, the visible cross section at the peak of the narrow resonances like J/ψ , $\Upsilon(1S)$ and so on is suppressed by a factor of order of $\Gamma_{tot}/\Delta E$ where ΔE is an energy spread. But, fortunately, the $\chi_{c1}(1P)$ resonance width equal to 0.88 MeV [1] is not small in comparison with energy spreads of current facilities, for example, $\Delta E \simeq 2\, MeV$ at BEPC (China), see [1]. Taking into account that the luminosity at BEPC [1] is equal to $10^{31}\, cm^{-2}s^{-1}$, $\Gamma_{tot}(\chi_{c1}(1P))/\Delta E \simeq 0.44$ and the cross section of the $\chi_{c1}(1P)$ production is equal to $1.14 \cdot 10^{-35}\, cm^2$, see Eq. (18), one can during an effective year (10^7 seconds) working produce 501 $\chi_{c1}(1P)$ states.

Note that such a number of $\chi_{c1}(1P)$ states gives 135 (27%) unique decays $\chi_{c1}(1P) \to \gamma J/\psi$.

The $c-\tau$ factories (luminosity $\sim 10^{33}\, cm^{-2}s^{-1}$) could produce several tens of thousands of the $\chi_{c1}(1P)$ states.

As for the $\chi_{b1}(1P)$ state, its width is unknown up to now [1]. Let us estimate it using the $\chi_{c1}(1P)$, J/ψ , $\Upsilon(1S)$ widths, and the quark model.

In the quark model

$$\Gamma\left({}^3S_1 \equiv V \to ggg\right) = \frac{40}{81\pi}\left(\pi^2 - 9\right)\frac{\alpha_s^3}{m_V^2}|R_S(0\, ,\, m_V)|^2 \tag{20}$$

One gets from Eqs. (13) and (20) that

$$\frac{\Gamma(A\to gq\bar{q})}{\Gamma(V\to ggg)}=$$
$$\frac{\Gamma_{tot}(A)}{\Gamma_{tot}(V)}\frac{BR(A\to hadrons)}{[BR(V\to hadrons)-BR(V\to virtual\,\gamma\to hadrons)]}=$$
$$99.4\frac{N}{3}\left(\frac{m_V}{m_A}\right)^2\left|\frac{R'_P(0\,,\,m_A)}{m_A R_S(0\,,\,m_V)}\right|^2\ln\frac{m_A R(m_A)}{2}\,. \tag{21}$$

So,

$$\frac{\Gamma_{tot}(\chi_{b1}(1P))}{\Gamma_{tot}(\Upsilon(1S))}=0.53\frac{\Gamma_{tot}(\chi_{c1}(1P))}{\Gamma_{tot}(J/\psi)}\left|\frac{R'_P(0\,,\,m_{\chi_{b1}(1P)})R_S(0\,,\,m_{J/\psi})}{R'_P(0\,,\,m_{\chi_{c1}(1P)})R_S(0\,,\,m_{\Upsilon(1S)})}\right|^2=$$
$$5.3\left|\frac{R'_P(0\,,\,m_{\chi_{b1}(1P)})R_S(0\,,\,m_{J/\psi})}{R'_P(0\,,\,m_{\chi_{c1}(1P)})R_S(0\,,\,m_{\Upsilon(1S)})}\right|^2\,. \tag{22}$$

Calculating Eq. (22) one used data from [1], $BR(\Upsilon(1S)\to hadrons)-BR(\Upsilon(1S)\to virtual\,\gamma\to hadrons)=0.83$, $BR(J/\psi\to hadrons)-BR(J/\psi\to virtual\,\gamma\to hadrons)=0.69$, $\Gamma_{tot}(\chi_{c1}(1P))/\Gamma_{tot}(J/\psi)=10$, and $L(m_{b1(1P)})/L(m_{c1(1P)})=2$ as in the foregoing.

The unknown factor in Eq. (22) depends on a model. In the Coulomb-like potential model it is

$$\left|\frac{R'_P(0\,,\,m_{\chi_{b1}(1P)})R_S(0\,,\,m_{J/\psi})}{R'_P(0\,,\,m_{\chi_{c1}(1P)})R_S(0\,,\,m_{\Upsilon(1S)})}\right|^2=\left(\frac{m_{\chi_{b1}(1P)}}{m_{\chi_{c1}(1P)}}\right)^5\left(\frac{m_{J/\psi}}{m_{\Upsilon(1S)}}\right)^3=6.2\,. \tag{23}$$

In the harmonic oscillator potential it is

$$\left|\frac{R'_P(0\,,\,m_{\chi_{b1}(1P)})R_S(0\,,\,m_{J/\psi})}{R'_P(0\,,\,m_{\chi_{c1}(1P)})R_S(0\,,\,m_{\Upsilon(1S)})}\right|^2=\left(\frac{m_{\chi_{b1}(1P)}}{m_{\chi_{c1}(1P)}}\right)^{2.5}\left(\frac{m_{J/\psi}}{m_{\Upsilon(1S)}}\right)^{1.5}=2.5\,. \tag{24}$$

To be conservative one takes Eq. (24). Thus one expects

$$\Gamma_{tot}(\chi_{b1}(1P))\simeq 13\Gamma_{tot}(\Upsilon(1S))\simeq 0.695\,MeV\,. \tag{25}$$

Let us estimate a number of the $\chi_{b1}(1P)$ states which can be produced at CESR (Cornell) [1]. Taking into account that luminosity at CESR is equal to $10^{32}\,cm^{-2}s^{-1}$, $\Delta E\simeq 6\,MeV$, $\Gamma_{tot}(\chi_{b1}(1P))/\Delta E\simeq 0.12$ and the cross section of the $\chi_{b1}(1P)$ production is equal to $4.8\cdot 10^{-35}\,cm^2$, see Eq. (19), one can during an effective year (10^7 seconds) working

produce 5622 $\chi_{b1}(1P)$ states. This number of the $\chi_{b1}(1P)$ states gives 1968 (35%) unique decays $\chi_{b1}(1P) \to \gamma\Upsilon(1S)$.

At VEPP-4M (Novosibirsk) [1] one can produce a few hundreds of the $\chi_{b1}(1P)$ states.

As for the b factories with luminosities $10^{33}\, cm^{-2}s^{-1}$ and $10^{34}\, cm^{-2}s^{-1}$ [1], they could produce tens and hundreds of thousands of the $\chi_{b1}(1P)$ states.

As for the $\chi_{b1}(2P)$ state, it is impossible to estimate its width by the considered way. The point is that the $\chi_{c1}(2P)$ is unknown up to now [1] (probably, this state lies above the threshold production of the $D\bar{D}^* + D^*\bar{D}$ heavy quarkonia). Nevertheless, it seems reasonable that $\Gamma_{tot}(\chi_{b1}(2P)) \sim 1\, MeV$ as in the case of the $\chi_{b1}(1P)$ state. That is why it is reasonable to believe, see Eq. (19), that there is a good chance to search for the direct production of the $\chi_{b1}(2P)$ state as in the case of the $\chi_{b1}(1P)$ state.

So, the current facilities give some chance to observe the $\chi_{c1}(1P)$ state production in the e^+e^- collisions and to study the production of the $\chi_{b1}(1P)$ and $\chi_{b1}(2P)$ states in the e^+e^- collisions in sufficient detail.

The $c-\tau$ and b factories would give possibilities to study in the e^+e^- collisions the $\chi_{c1}(1P)$ state production in sufficient detail and the production of the $\chi_{b1}(1P)$ and $\chi_{b1}(2P)$ states in depth. Probably, it is possible to observe the $\chi_{c1}(2P)$ state production at the $c-\tau$ factories.

The fine effects considered above are essential not only to the understanding of the quark model but can be used for identification of the $\chi_{b1}(1P)$ and $\chi_{b1}(2P)$ states because the angular momentum J of the states named as $\chi_{b1}(1P)$ and $\chi_{b1}(2P)$ needs confirmation [1].

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